8-2 Special Right Triangles

Common Core State Standards

G-SRT.C.8 Use . . . the Pythagorean Theorem to solve right triangles in applied problems.

MP 1, MP 3, MP 4

Objective To use the properties of 45°-45°-90° and 30°-60°-90° triangles

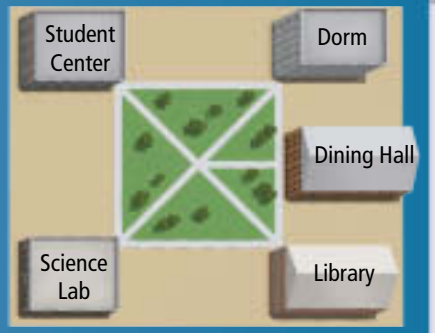
There are a lot of similar right triangles here. In the lesson, you'll learn a shortcut for finding some of these distances.

MATHEMATICAL PRACTICES

Getting Ready!

This map of part of a college campus shows a square "quad" area with walking paths. The distance from the dorm to the dining hall is 150 yd.

Suppose you go from your dorm to the dining hall, to the science lab, to your dorm, to the student center, to the library, and finally back to your dorm. To the nearest tenth, how far do you walk? Justify your answer. (Assume you always take the most direct routes and stay on the paths.)



The Solve It involves triangles with angles 45°, 45°, and 90°.

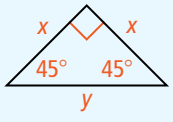
Essential Understanding Certain right triangles have properties that allow you to use shortcuts to determine side lengths without using the Pythagorean Theorem.

The acute angles of a right isosceles triangle are both 45° angles. Another name for an isosceles right triangle is a 45°-45°-90° triangle. If each leg has length x and the hypotenuse has length y, you can solve for y in terms of x.

$x^2 + x^2 = y^2$ Use the Pythagorean Theorem.

$2x^2 = y^2$ Simplify.

$x\sqrt{2} = y$ Take the positive square root of each side.



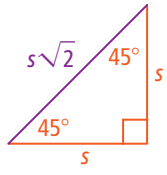
You have just proved the following theorem.

take note

Theorem 8-5 45°-45°-90° Triangle Theorem

In a 45°-45°-90° triangle, both legs are congruent and the length of the hypotenuse is $\sqrt{2}$ times the length of a leg.

hypotenuse = $\sqrt{2}$ • leg



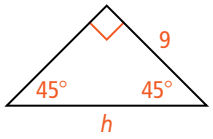
Problem 1 Finding the Length of the Hypotenuse

What is the value of each variable?

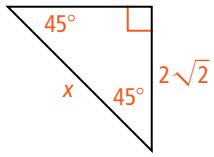
Think

Why is only one leg labeled?
A 45°-45°-90° triangle is a right isosceles triangle, so the legs have equal lengths.

A



B



hypotenuse $= \sqrt{2} \cdot$ leg	45°-45°-90° △ Theorem	hypotenuse $= \sqrt{2} \cdot$ leg
$h = \sqrt{2} \cdot 9$	Substitute.	$x = \sqrt{2} \cdot 2\sqrt{2}$
$h = 9\sqrt{2}$	Simplify.	$x = 4$

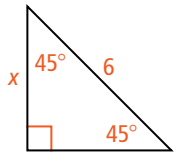
Got It? **1.** What is the length of the hypotenuse of a 45°-45°-90° triangle with leg length $5\sqrt{3}$?

Problem 2 Finding the Length of a Leg

Think

Can you eliminate any of the choices?
The variable x represents the length of a leg. Since the hypotenuse is the longest side of a right triangle, $x < 6$. You can eliminate choices C and D.

Multiple Choice **What is the value of x?**



(A) 3
(B) $3\sqrt{2}$
(C) 6
(D) $6\sqrt{2}$

hypotenuse $= \sqrt{2} \cdot$ leg	45°-45°-90° Triangle Theorem
$6 = \sqrt{2} \cdot x$	Substitute.
$x = \frac{6}{\sqrt{2}}$	Divide each side by $\sqrt{2}$.
$x = \frac{6}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}}$	Multiply by a form of 1 to rationalize the denominator.
$x = \frac{6\sqrt{2}}{2}$	Simplify.
$x = 3\sqrt{2}$	Simplify.

The correct answer is B.

Got It? **2. a.** The length of the hypotenuse of a 45°-45°-90° triangle is 10. What is the length of one leg?

b. Reasoning In Problem 2, why can you multiply $\frac{6}{\sqrt{2}}$ by $\frac{\sqrt{2}}{\sqrt{2}}$?

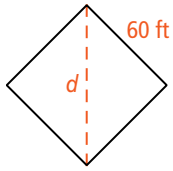
When you apply the 45°-45°-90° Triangle Theorem to a real-life example, you can use a calculator to evaluate square roots.

Problem 3 Finding Distance

Think

How do you know that *d* is a hypotenuse?
The diagonal *d* is part of two right triangles. The hypotenuse of a right triangle is always opposite the 90° angle. So *d* must be a hypotenuse.

Softball **A high school softball diamond is a square. The distance from base to base is 60 ft. To the nearest foot, how far does a catcher throw the ball from home plate to second base?**



The distance d is the length of the hypotenuse of a 45°-45°-90° triangle.

$d = 60\sqrt{2}$ hypotenuse $= \sqrt{2} \cdot$ leg

$d \approx 84.85281374$ Use a calculator.

The catcher throws the ball about 85 ft from home plate to second base.

Got It? **3.** You plan to build a path along one diagonal of a 100 ft-by-100 ft square garden. To the nearest foot, how long will the path be?

Another type of special right triangle is a 30°-60°-90° triangle.

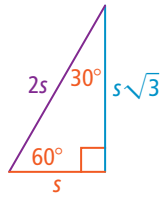
take note

Theorem 8-6 30°-60°-90° Triangle Theorem

In a 30°-60°-90° triangle, the length of the hypotenuse is twice the length of the shorter leg. The length of the longer leg is $\sqrt{3}$ times the length of the shorter leg.

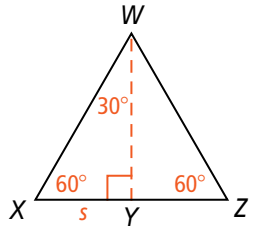
hypotenuse $= 2 \cdot$ shorter leg

longer leg $= \sqrt{3} \cdot$ shorter leg



Proof **Proof of Theorem 8-6: 30°-60°-90° Triangle Theorem**

For equilateral $\triangle WXZ$, altitude $\overline{WY}$ bisects $\angle W$ and is the perpendicular bisector of $\overline{XZ}$. So, $\overline{WY}$ divides $\triangle WXZ$ into two congruent 30°-60°-90° triangles.

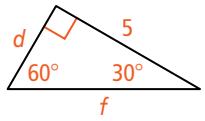


Thus, $XY = \frac{1}{2}XZ = \frac{1}{2}XW$, or $XW = 2XY = 2s$.

$XY^2 + YW^2 = XW^2$	Use the Pythagorean Theorem.
$s^2 + YW^2 = (2s)^2$	Substitute s for XY and $2s$ for XW.
$YW^2 = 4s^2 - s^2$	Subtract s^2 from each side.
$YW^2 = 3s^2$	Simplify.
$YW = s\sqrt{3}$	Take the positive square root of each side.

You can also use the 30°-60°-90° Triangle Theorem to find side lengths.

Problem 4 Using the Length of One Side



Algebra **What is the value of *d* in simplest radical form?**

Think

In a 30°-60°-90° triangle, the leg opposite the 60° angle is the longer leg. So *d* represents the length of the shorter leg. Write an equation relating the legs.

Write

$\text{longer leg} = \sqrt{3} \cdot \text{shorter leg}$

$5 = d\sqrt{3}$

Divide each side by $\sqrt{3}$ to solve for *d*.

$d = \frac{5}{\sqrt{3}}$

The value of *d* is not in simplest radical form because there is a radical in the denominator. Multiply *d* by a form of 1.

$\frac{5}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{5\sqrt{3}}{3}$

So $d = \frac{5\sqrt{3}}{3}$.

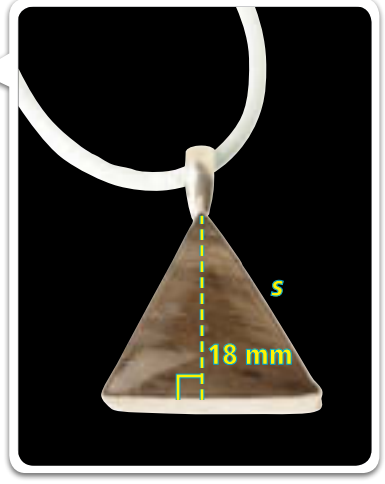
Got It? **4.** In Problem 4, what is the value of *f* in simplest radical form?

Problem 5 Applying the 30°-60°-90° Triangle Theorem

Plan

How does knowing the shape of the pendants help?
Since the triangle is equilateral, you know that an altitude divides the triangle into two congruent 30°-60°-90° triangles.

Jewelry Making **An artisan makes pendants in the shape of equilateral triangles. The height of each pendant is 18 mm. What is the length *s* of each side of a pendant to the nearest tenth of a millimeter?**



The hypotenuse of each 30°-60°-90° triangle is *s*. The shorter leg is $\frac{1}{2}s$.

$18 = \sqrt{3}\left(\frac{1}{2}s\right)$ longer leg = $\sqrt{3}$ • shorter leg

$18 = \frac{\sqrt{3}}{2}s$ Simplify.

$\frac{2}{\sqrt{3}} \cdot 18 = s$ Multiply each side by $\frac{2}{\sqrt{3}}$.

$s \approx 20.78460969$ Use a calculator.

Each side of a pendant is about 20.8 mm long.

Got It? **5.** Suppose the sides of a pendant are 18 mm long. What is the height of the pendant to the nearest tenth of a millimeter?

Lesson Check

Do you know HOW?

What is the value of x? If your answer is not an integer, express it in simplest radical form.

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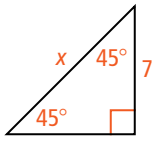

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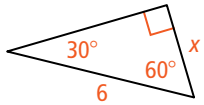

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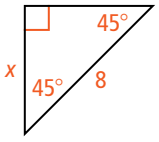

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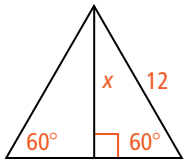
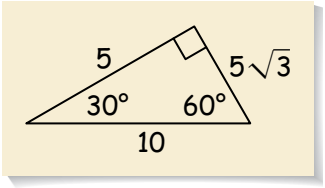


Do you UNDERSTAND? MATHEMATICAL PRACTICES

5. Error Analysis Sandra drew the triangle below. Rika said that the labeled lengths are not possible. With which student do you agree? Explain.



6. Reasoning A test question asks you to find two side lengths of a 45°-45°-90° triangle. You know that the length of one leg is 6, but you forgot the special formula for 45°-45°-90° triangles. Explain how you can still determine the other side lengths. What are the other side lengths?

Practice and Problem-Solving Exercises MATHEMATICAL PRACTICES

A Practice

Find the value of each variable. If your answer is not an integer, express it in simplest radical form.

See Problems 1 and 2.

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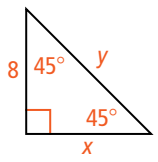

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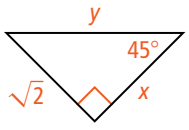

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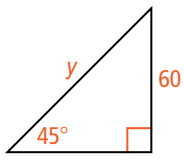

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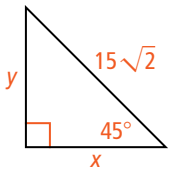

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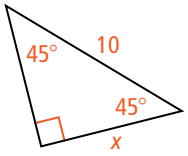

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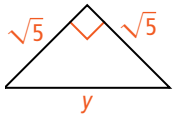


13. Dinnerware Design What is the side length of the smallest square plate on which a 20-cm chopstick can fit along a diagonal without any overhang? Round your answer to the nearest tenth of a centimeter.

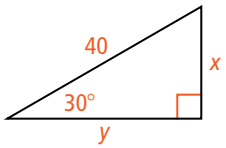
See Problem 3.

14. Aviation The four blades of a helicopter meet at right angles and are all the same length. The distance between the tips of two adjacent blades is 36 ft. How long is each blade? Round your answer to the nearest tenth of a foot.

Algebra **Find the value of each variable. If your answer is not an integer, express it in simplest radical form.**

See Problems 4 and 5.

15.



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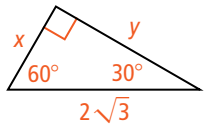

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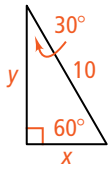

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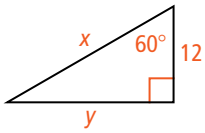

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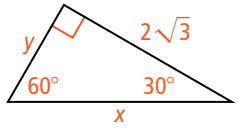

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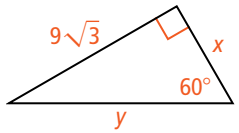
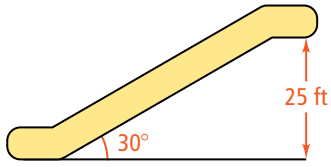


STEM **21. Architecture** An escalator lifts people to the second floor of a building, 25 ft above the first floor. The escalator rises at a 30° angle. To the nearest foot, how far does a person travel from the bottom to the top of the escalator?



STEM **22. City Planning** Jefferson Park sits on one square city block 300 ft on each side. Sidewalks across the park join opposite corners. To the nearest foot, how long is each diagonal sidewalk?

B Apply

Algebra **Find the value of each variable. If your answer is not an integer, express it in simplest radical form.**

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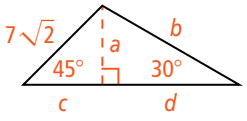

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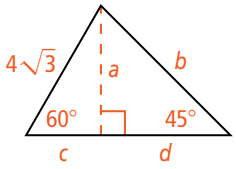

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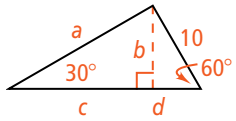

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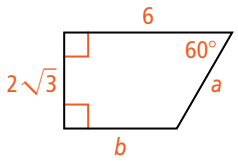

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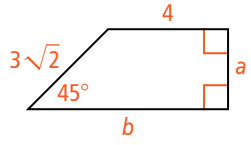

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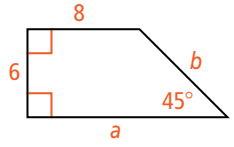
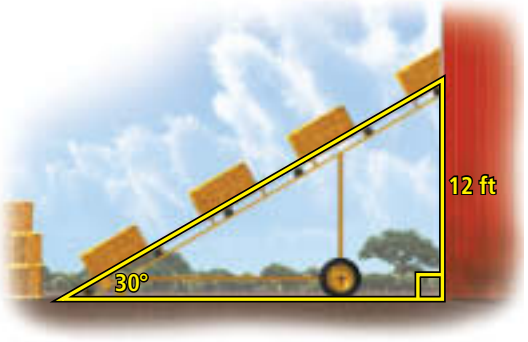


29. Think About a Plan A farmer's conveyor belt carries bales of hay from the ground to the barn loft. The conveyor belt moves at 100 ft/min. How many seconds does it take for a bale of hay to go from the ground to the barn loft?

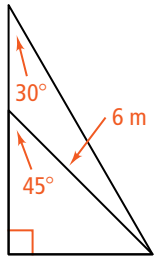
- Which part of a right triangle does the conveyor belt represent?
- You know the speed. What other information do you need to find time?
- How are minutes and seconds related?



30. House Repair After heavy winds damaged a house, workers placed a 6-m brace against its side at a 45° angle. Then, at the same spot on the ground, they placed a second, longer brace to make a 30° angle with the side of the house.

a. How long is the longer brace? Round to the nearest tenth of a meter.

b. About how much higher does the longer brace reach than the shorter brace?



31. Open-Ended Write a real-life problem that you can solve using a 30°-60°-90° triangle with a 12-ft hypotenuse. Show your solution.

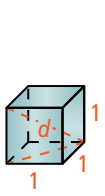
32. Constructions Construct a 30°-60°-90° triangle using a segment that is the given side.

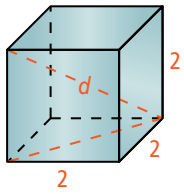
a. the shorter leg **b.** the hypotenuse **c.** the longer leg

Challenge

33. Geometry in 3 Dimensions Find the length d, in simplest radical form, of the diagonal of a cube with edges of the given length.

a. 1 unit **b.** 2 units **c.** s units





Standardized Test Prep

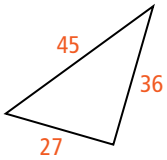
SAT/ACT

34. The longer leg of a 30°-60°-90° triangle is 6. What is the length of the hypotenuse?

Ⓐ $2\sqrt{3}$ Ⓑ $3\sqrt{2}$ Ⓒ $4\sqrt{3}$ Ⓓ 12

35. Which triangle is NOT a right triangle?

Ⓕ



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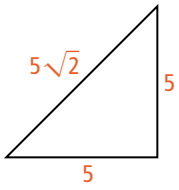

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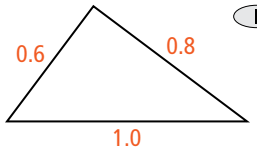

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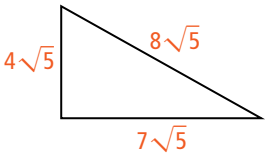

36. Suppose p is false and q is true. Which statement is NOT true?

Ⓐ $p \rightarrow q$ Ⓑ $\sim q \vee (p \wedge q)$ Ⓒ $p \vee q$ Ⓓ $(p \vee q) \wedge \sim p$

Short Response

37. In right $\triangle ABC$, $\angle C$ is the right angle and $\overline{CD}$ is the altitude drawn to the hypotenuse. If $AD = 3$ and $DB = 9$, what is AC? Show your work.

Mixed Review

38. A right triangle has a 6-in. hypotenuse and a 5-in. leg. Find the length of the other leg in simplest radical form.

See Lesson 8-1.

39. An isosceles triangle has 20-cm legs and a 16-cm base. Find the length of the altitude to the base in simplest radical form.

Get Ready! **To prepare for Lesson 8-3, do Exercises 40–43.**

Algebra **Solve each proportion.**

See Lesson 7-1.

40. $\frac{x}{3} = \frac{4}{7}$ **41.** $\frac{6}{11} = \frac{x}{9}$ **42.** $\frac{8}{15} = \frac{4}{x}$ **43.** $\frac{5}{x} = \frac{7}{12}$